Rank of a matrix is a systematic technique to eliminate the linearly dependent vectors.

The elementary operations are consequences of solving a system of linear equations

The row operations are

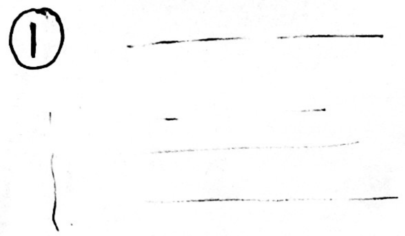
#1. $R_m \leftrightarrow R_n$ (YOU ARE ALLOWED TO INTERCHANGE ANY TWO ROWS)

#2. $R_m \rightarrow k R_m$ $(k \neq 0)$ (YOU ARE ALLOWED TO MULTIPLY ANY ROW BY A NON ZERO CONSTANT)

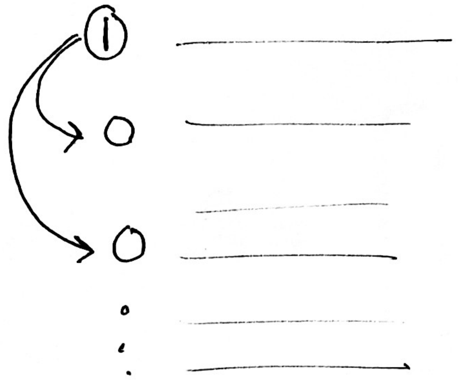
#3. $R_m \rightarrow R_m + k R_n$ (YOU ARE ALLOWED TO ADD a Row with a multiple of another Row)

NOTE - The column operations are obtained by simply changing R in to C

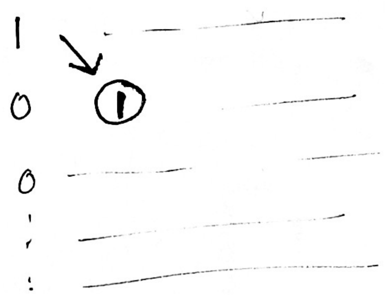
#1. NORMAL FORM



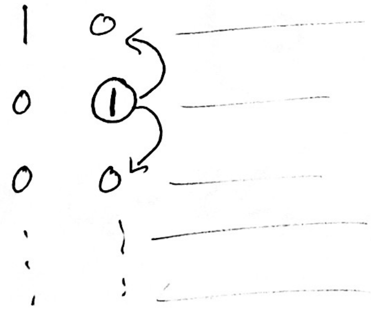
Make $a_{11} \rightarrow 1$ using ERT or ECT



Using this 1 change all the elements below 1 in to zeros



Try to make the diagonal element 1



Make all the elements above & below 1 in to zeros

Repeat the process, till you can see an Indentity submatrix and the remaining zeros

RANK = Order of the Identity Submatrix

Find the rank of the following matrices Using Normal Form.

1. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$R_2 \to R_2 - 4R_1, \quad R_3 \to R_3 - 7R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1, \quad C_3 \to C_3 - 3C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix}$$

ie. $A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{bmatrix}$ $R_2 \to R_2 \times \frac{1}{-3}$

Now $R_3 \to R_3 + 6R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - 2C_2$

$$\therefore A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

ie. $A \sim \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$ $\therefore$ RANK $= 2$